THE MEASUREMENT OF TECHNICAL EFFICIENCY WITH PARAMETRIC DISTANCE FUNCTIONS: AN APPLICATION TO SPANISH PISA RESULTS

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ABSTRACT

The aim of this paper is to propose a parametric stochastic distance function approach to measure educational efficiency. This work explicitly considers that education is a multi-input multi-output process subject to inefficient behaviors that can be identified at student level. Students use self and school inputs in order to transform them in academic results; therefore a translog distance function allows us to calculate several aspects of the educational technology; mainly output elasticities with respect to inputs, and to estimate the importance of efficiency slacks. The paper presents an empirical application of this model using Spanish data from the Programme for International Student Assessment (PISA) conducted in 2000 by the OECD. The results provide insights over how student's background, peer-group and school characteristics, mainly class sizes, match with educational outputs: reading and mathematics test scores. Findings also suggest that once taking into account these student characteristics and school inputs there is no statistically significant difference in performances across schools regarding public-private ownership.

Key words: educational production function, efficiency, parametric distance function.

JEL Classification: I21, D24, C4

1. Introduction

Since the early-1960s a wide number of studies (*e.g.* Coleman, 1966; Summers *et al.* 1977, Hanushek, 1986) have sought the relationship between school inputs, students' background and achievement at school. Despite all the research devoted to this issue the well-known "Does school matter?-Does money matter?" debate remains open yet. Many evidences were found that student's education takes place both, at home and at school. However, the way that students' self characteristics, home, peer-group and school match with educational outputs continues to be widely unknown and this is a serious drawback for policy-makers taking decisions about the allocation of the scarce public resources devoted to education.

We can summarize main reasons put forward in the literature why empirical work does not find systematic relationships between school inputs and outputs. First, education is a high complex process with variables like organization or non-monetary inputs implied in production [Vandenberghe, 1999]. Second, the inconsistency of the use of Cobb-Douglas specifications for the estimation of the educational production functions [see for example Eide and Showalter (1998), Figlio (1999)]. Third, most production function studies in economics of education does not consider the theoretical potential role of the efficiency component. And, last but not least, in empirical research typically student's results are aggregated at school or district levels imposing a limitation in order to disentangle the effect of student's self background from peer-group and school inputs on students' achievement.

In order to overcome the underlined difficulties, we propose the use of frontier analysis techniques, more precisely a parametric stochastic distance function, and explicitly consider that education is a process in which students use self and school inputs in order to transform them in academic results, subject to inefficient behaviors that can be identified at student level. To illustrate the potentialities of the approach proposed here, we provide an application to Spanish's educational data from PISA conducted in 2000 by the OECD. Moreover, we investigate differences in students' performances across Spanish public and private schools.

The paper is organized as follows. Section 2 provides a rough overview of educational production functions and presents the parametric stochastic distance function. Section 3 is devoted to describe data and provides results of our empirical analysis. Finally, the paper ends with a summary and explores directions for further research.

2. Estimating an educational production function through distance functions.

In most studies a common conceptual framework for estimating the educational production function might take the following form:

$$A_{is} = f\left(B_{is}, S_s, P_{is}, I_{is}\right) \qquad (1)$$

Where A_{is} equals the achievement of student *i* at school *s*, B_{is} is student's background, S_s are school inputs, P_{is} denotes the peer-group effect, and I_{is} are student innate abilities. Most of times, equation (1) is estimated at school levels. This analysis typically aggregates students' achievements and inputs belonging to each school in average by school, or even by school district when some non-controllable inputs are not observable at school level.

In this paper we propose to use parametric stochastic distance functions at student level in order to go beyond in the analysis of production functions in education. For this purpose equation (1) becomes:

$$D_{is} = g\left(A_{is}, B_{is}, S_s, P_{is}\right) . I_i \qquad (2)$$

Where g represents the best practice technology used in the transformation of educational inputs in outputs, and D_{is} is the distance that separates each student from the technological boundary. Unobservable students innate abilities, I_i, are assumed to be randomly distributed in the population and influence individual performances in a multiplicative way. This simple transformations place naturally the empirical estimation of equation (2) in the framework of parametric stochastic frontier analysis (SFA) that, under specific distributional assumptions, allows disentangling random effects from efficiency (distance to the frontier).

The quality of public schools has recently been under scrutiny with voices claming for reforming the Spanish Law for Quality in Schooling (*Ley Orgánica de Calidad de la Enseñanza*). As in most other countries, there are three possibilities in financing a school in Spain: private schools, private schools financed by a vouchers system and public schools. The argument asking for more private schools and for more public expenditure in education monitored by private hands is usually based on aggregate results, like those we obtained from PISA 2000 presented in Table 1.

Table 1: Mathematics and reading scores by school type in Spain, PISA 2000

School type	N	Mean	Standard deviation	Minimum	Maximum
Mathematics scores					
Private, government independent	16	539.09	34.23	468.22	589.44
Private, government dependent	56	510.11	39.80	401.97	577.87
Government	113	495.27	37.42	338.60	573.86
All	185	503.55	39.83	338.60	589.44
Reading scores					
Private, government independent	16	557.15	29.03	499.69	616.65
Private, government dependent	56	529.68	37.25	439.78	596.38
Government	113	513.27	38.01	388.43	582.82
All	185	522.03	39.18	388.43	616.65

Note: Mean differences are statistically significant, at 95% level, with F-test=10.5 and 11.5 for mathematics and reading, respectively. We cannot reject that variances are distributed homogenously, at 95% level, with Levene's test=0.169 and 0.587 for mathematics and reading, respectively.

A *naïve* interpretation of the descriptive results presented in Table 1 would bring us to conclude to a higher performance of private schools in terms of average scores, but also in terms of equity. The study presented here, based in the estimation of a parametric stochastic distance function, can shed light to confirm or to refuse this appreciation.

2.1. The parametric stochastic distance function approach

Defining a vector of inputs $x = (x_1, \ldots, x_K) \in \Re^{K+}$ and a vector of outputs $y = (y_1, \ldots, y_M) \in \Re^{M+}$ the feasible multi-input multi-output production technology can be defined using de output possibility set $P(x)$ which can be produced using the input vector x:

P(x) = {y: x can produce *y)* that is assumed to satisfy the set of axioms depicted in Färe and Primont (1995). This technology can also be defined as the output distance function proposed by Shephard (1970):

$$D_O(x,y) = inf\{\theta : \theta > 0, (x, y/\theta) \in P(x)\}$$

If $D_O(x,y) \leq 1$ then (x,y) belongs to the production set *P*(*x*). In addition $D_O(x,y) = 1$ if *y* is located on the outer boundary of the output possibility set. In order to estimate the distance function in a parametric setting a translog functional form is assumed. According with Coelli and Perelman (1999) this specification fulfils a set of desirable characteristics: flexible, easy to derive and allowing the imposition of homogeneity. The translog distance function specification herein adopted for the case of *K* inputs and *M* outputs is:

$$ln\, D_{Oi}(x,y) = \alpha_0 + \sum_{m=1}^{M} \alpha_m \, ln\, y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \, ln\, y_{mi} \, ln\, y_{ni} + \sum_{k=1}^{K} \beta_k \, ln\, x_{ki} \quad (3)$$

$$+ \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \, ln\, x_{ki} \, ln\, x_{li} + \sum_{k=1}^{K}\sum_{m=1}^{M} \delta_{km} \, ln\, x_{ki} \, ln\, y_{mi} \, , \quad i = 1,2,\ldots, N,$$

where *i* denotes the i^{th} unit in the sample. In order to obtain the production frontier surface we set $D_O(x,y) = 1$ which implies $ln\, D_O(x,y) = 0$. The parameters of the above distance function must satisfy a number of restrictions, among them symmetry and homogeneity of degree + 1 in outputs. According with Lovell *et al*. (1994), to normalize the output distance function by one of the outputs is equivalent to imposing homogeneity of degree +1. Rearranging terms the function above can be rewritten as follows:

$$-ln(y_{Mi}) = TL(x_i, y_i / y_{Mi}, \alpha, \beta, \delta) - ln\, D_{Oi}(x,y), \quad i = 1, 2, \ldots, N, \quad (4)$$

where $-ln\, D_{Oi}(x,y)$ corresponds to the radial distance function from the boundary. Hence we can set $u = -ln\, D_{Oi}(x,y)$ and add up a term v_i capturing for noise to obtain the Battese and Coelli (1988) version of the SFA model proposed by Aigner *et al*. (1977).

$$-\ln\left(y_{Mi}\right) = TL(x_i, y_i / y_{Mi}, \alpha, \beta, \delta) + \varepsilon_i, \quad \varepsilon_i = v_i + u_i, \tag{5}$$

where $u = -\ln D_{Oi}\left(x, y\right)$, the distance to the boundary set, is a negative random term assumed to be independently distributed as truncations at zero of the $N\left(0, \sigma_u^2\right)$ distribution, and the v_i term is assumed to be a two-sided random (stochastic) disturbance designated to account for statistical noise and distributed *iid* $N\left(0, \sigma_v^2\right)$. But these are the normal assumptions given to the u_i and v_i error terms in frontier analysis literature dealing with firms' technical efficiency in production. What is the interpretation we can give to these error terms in the particular case of student performance treated here? We think that they allow for a straight interpretation. On the one hand, the stochastic term v_i is expected to catch the non observed student characteristics, mainly innate abilities, but also aptitudes to perform tests and luck, as well as family specific circumstances, e. g. parents' labor force status, potentially affecting students' results but not captured by the model. All of them are assumed to be distributed normally random in the population. On the other hand, the distance function term u_i is expected to catch students and teachers' efforts and motivation as well as school performances not explained by input endowments included in the distance function.

3. An application to Spanish secondary schools

3.1. Data

In our empirical analysis we use data from PISA conducted in 2000 by the OECD. PISA tested students in the subjects of reading, mathematics and science. Given that the target 15-years-old population tends to be enrolled in two grades, we selected for this study upper 10th grade students. Summing up, the analysis is based in a homogenous population composed of 2449 students attending 185 different schools, which in year 2000 completed the tests.

We consider two outputs: the students' scores obtained in the international mathematics and reading tests. As reported in Table 2, average reading scores are higher and at the same time less widely distributed than mathematics scores. Two school inputs were selected. On one hand, the *computers/students* ratio corresponding to the total number of computers in the school divided by the total enrollment and, on the other hand, the *teachers/students* ratio corresponding to the total teaching staff divided by the total school enrollment.

Table 2: Descriptive statistics: outputs and inputs at student level

Outputs and inputs	Variable	Mean	Std. dev.	Minimum	Maximum
Outputs					
Mathematics score	y_1	505.3	82.9	202.1	815.9
Reading score	y_2	524.0	74.3	241.4	741.9
Inputs					
School					
Computers / 100 students	x_1	6.36	4.10	0.90	31.00
Teachers / 100 students	x_2	7.59	2.36	3.62	17.67
Background					
Mother education	x_3	2.79	0.78	1.00	4.00
Father education	x_4	2.89	0.82	1.00	4.00
Cultural activities	x_5	2.54	1.17	1.00	5.00
Cultural possessions	x_6	3.08	0.99	1.00	4.00
Time spent on homework	x_7	3.37	0.81	1.00	4.00
Peer-Group					
Average mother education	x_8	2.88	0.43	1.90	4.00

We consider five students background inputs. All of these variables are represented by indices that summarize the answers given by students to a series of related questions. *Mother* and *father education* correspond to the International Standard Classification of Education (ISCED, OECD, 1999). Together with *Cultural activities*, *cultural possessions* and *time spent on homework* we introduce a variable to control for potential *peer-group* effects. The variable considered here is the average mothers' education measured at the school level.

3.2. Model estimation

A parametric output distance function was estimated assuming a stochastic translog technology, as indicated in Section 2. Homogeneity of degree +1 was imposed by selecting one of the outputs, the students' scores in mathematics y_1 as the dependent variable, and the ratio

y_2/y_1 as explanatory variable. However, for presentation purposes, in Table 3 the parameters corresponding to y_1 are reported, as calculated by application of the homogeneity condition.

Table 3: Parametric output distance function estimations

Variables and parameters			t-ratio	Variables and parameters			t-ratio
Intercept	α_0	0.1429	19.52	Inputs (Cont.)			
Outputs				(ln x_1)(ln x_5)	β_{15}	-0.0188	1.98
ln y_1 (mathematics score)	α_1	-0.3757		(ln x_1)(ln x_6)	β_{16}	0.0152	1.28
ln y_2 (reading score)	α_2	-0.6243	41.45	(ln x_1)(ln x_7)	β_{17}	0.0166	1.01
(ln y_1)^2	α_{11}	-1.5089		(ln x_1)(ln x_8)	β_{18}	0.0857	2.26
(ln y_2)^2	α_{22}	-1.5089	17.38	(ln x_2)(ln x_3)	β_{23}	0.0601	1.69
(ln y_1)(lny_2)	α_{12}	1.5089		(ln x_2)(ln x_4)	β_{24}	-0.0616	1.69
Inputs				(ln x_2)(ln x_5)	β_{25}	0.0073	0.42
ln x_1 (computers/students)	β_1	0.0002	0.05	(ln x_2)(ln x_6)	β_{26}	0.0159	0.75
ln x_2 (teachers/students)	β_2	0.0046	0.54	(ln x_2)(ln x_7)	β_{27}	-0.0017	0.06
ln x_3 (mother education)	β_3	0.0357	3.35	(ln x_2)(ln x_8)	β_{28}	-0.1638	2.42
ln x_4 (father education)	β_4	0.0214	1.90	(ln x_3)(ln x_4)	β_{34}	0.0570	1.96
ln x_5 (cultural activities)	β_5	0.0414	7.79	(ln x_3)(ln x_5)	β_{35}	-0.0005	0.03
ln x_6 (cultural possessions)	β_6	0.0288	2.94	(ln x_3)(ln x_6)	β_{36}	-0.0185	0.75
ln x_7 (homework)	β_7	0.0209	1.77	(ln x_3)(ln x_7)	β_{37}	0.0063	0.22
ln x_8 (peer-group)	β_8	0.1497	7.81	(ln x_3)(ln x_8)	β_{38}	-0.0240	0.30
(ln x_1)^2	β_{11}	-0.0124	1.17	(ln x_4)(ln x_5)	β_{45}	0.0074	0.40
(ln x_2)^2	β_{22}	-0.1620	3.11	(ln x_4)(ln x_6)	β_{46}	0.0162	0.70
(ln x_3)^2	β_{33}	-0.0930	2.01	(ln x_4)(ln x_7)	β_{47}	-0.0121	0.43
(ln x_4)^2	β_{44}	-0.0250	0.59	(ln x_4)(ln x_8)	β_{48}	-0.0879	1.15
(ln x_5)^2	β_{55}	0.0576	2.72	(ln x_5)(ln x_6)	β_{56}	-0.0066	0.54
(ln x_6)^2	β_{66}	0.0189	0.70	(ln x_5)(ln x_7)	β_{57}	-0.0288	1.82
(ln x_7)^2	β_{77}	-0.0015	0.04	(ln x_5)(ln x_8)	β_{58}	0.0293	0.79
(ln x_8)^2	β_{88}	-0.0204	0.09	(ln x_6)(ln x_7)	β_{67}	-0.0322	1.86
(ln x_1)(ln x_2)	β_{12}	0.0656	3.70	(ln x_6)(ln x_8)	β_{68}	0.0322	0.68
(ln x_1)(ln x_3)	β_{13}	0.0079	0.43	(ln x_7)(ln x_8)	β_{78}	0.0323	2.86
(ln x_1)(ln x_4)	β_{14}	-0.0106	0.58				
Other ML parameters	γ	0.8067	30.84	Expected mean efficiency		0.8821	
	σ^2	0.0286	19.17				

Notes: Underlined parameters are calculated by applying imposed homogeneity conditions; all of the parameters are multiplied by -1.

As usually for the estimation of *translog* functions, the original variables, $y_m(m=1,2)$ and $x_k(k=1,...,8)$, were transformed in deviations to mean values. Therefore, first-order parameters in Table 3 must be interpreted as distance function partial elasticities at mean values. Moreover, all first order coefficients on inputs are positive, which indicates that, at least at mean values, students' performances increase (distance functions decrease) when inputs increase. All these coefficients are significant, with the only exception of both school inputs: *computers/students* and *teachers/students* ratios. Any general conclusion can be however drawn from these results without taking into account second order coefficients. Several of them are statistical significant, *e.g.* β_{22}, β_{12} and β_{23}, which correspond to the *teachers/students* ratio in its quadratic form and in interaction with other variables.

In our case, a simpler Cobb-Douglas production function estimation would be certainly unable to find out cross effects between school inputs, themselves or combined with students' background and peer-group inputs, and the conclusion would be "school does not matter". Therefore, one of the mayor advantages of parametric output distance function analysis at student level is that it can provide additional insights into the educational production process overcoming at the same time model misspecification problems.

3.4. Does school ownership matters?

Finally, we focus our attention on school ownership. Table 4 report efficiency results by school type that must be compared with descriptive statistics presented in Table 1.

Table 4: Efficiency and school ownership

School type	N	Mean	Standard deviation	Minimum	Maximum
Private, government independent	16	0.890	0.028	0.83	0.93
Private, government dependent	56	0.889	0.022	0.84	0.92
Government	113	0.886	0.029	0.75	0.94
All	185	0.887	0.027	0.75	0.94

Note: Mean differences are not statistically significant, at 95% level, with F-test=0.236. Variances are distributed homogenously, at 95% level, with Levene's test=1.051.

What can we learn from this comparison? Overall, that once school inputs, students background and peer-group, are taken into account, the observed differences across schools, distinguished upon ownership, vanish. Students attending private schools obtained better results as direct consequence of more favorable conditions: better family background, peer-groups and school inputs. Furthermore, a well-known selection process is at work in Spain like in other countries that offers that choice between public and private schools. As a consequence, public schools face up a higher percentage of students with less favorable background, e. g. foreigner population with language difficulties and special needs. As the estimated parametric stochastic distance function model take into account these and other students' characteristics, public schools take as benchmark this less favorable context and, as expected, their efficiency scores are better than when directly compared to private schools scores in Table 1.

4. Concluding remarks

In this paper we propose the use of frontier analysis techniques, more precisely a parametric stochastic distance function. We applied this methodology to the Spanish case using the 15-years-old student's tests scores and background data available from the PISA Project conducted by the OECD in 2000. The main results of this study can be summarized as follows:

We were particularly interested on a comparison between public and private school scores. The results show that in the case of Spain, the observed differences in favor of private schools are mainly accounted by differences in school inputs, students' background and peer-group characteristics considered as production factors in the education process.

The analysis conducted here reveals that school inputs matter but that their effects are better captured with a *translog* specification, a non-linear second order approximation, which takes into account the multi-output multi-input nature of education production.

To sum up, we think that the conceptual framework presented provides an appealing methodology for enhancing our understanding of the educational process. Furthermore, the measurement of educational technology and efficiency at student level shed light about how to distribute school vouchers. From this point of view, we think that public schools have an important role to play in the allocation of school inputs according with student background.

References

Aigner, D. J. Lovell, C. A. K. and Schmidt, P. (1977) Formulation and estimation of stochastic production function models, *Journal of Econometrics*, 6, 21-37.

Battese, G. E. and Coelli, T. J. (1988). Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data, *Journal of Econometrics*, 38, 387-399.

Coelli, T. and Perelman, S. (1999). A comparison of parametric and non-parametric distance functions: with application to European railways. *European Journal of Operational Research*, 117, 326-339.

Coleman, J. S. *et al.* (1966), Equality of educational opportunity (Washington, DC: U.S. GPO).

Eide, E. and Showalter, M. H. (1998), The effect of school quality on student performance: A quantile regression approach. *Economics Letters* 58, pp. 345-350.

Färe, R. and Primont, D. (1995). Multi-output production and duality: theory and applications. Kluwer Academic Publishers, Boston.

Figlio, D.N. (1999). Functional form and the estimated effects of school resources. *Economics of Education Review* 18, 241-252.

Hanushek, E. A. (1986). The economics of schooling, *Journal of Economic Literature* 24 (3), pp. 1141-1171.

Lovell, C. A. K., Richardson, S., Travers, P. and Wood, L. L. (1994). Resources and functionings: a new view of inequality in Australia, in Models and Measurement of Welfare and Inequality (Ed.) W. Eichhorn, Springer-Verlag, Berlin, 787-807.

OECD (1999). Classifying educational programmes: Manual for ISCED-97 implementation in OECD countries, Paris.

Shepard, R. W. (1970), Theory of Cost and Production Functions, Princeton University Press, Princeton, NY.

Summers, A. A. and Wolfe, B. L. (1977) Do schools make a difference? *American Economic Review* 67 (4), pp. 639-652.

Vandenberghe, V. (1999), Economics of education. The need to go beyond human capital theory and production-function analysis. *Educational Studies*, 25 (2), pp. 129-143.

Worthington, A. C. (2001). An empirical survey of frontier efficiency measurement techniques in education. *Education Economics*, Vol. 9, Nº 3.